

Pattern formation of frictional fingers in a gravitational potential

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Abstract

Aligned finger structures, with a characteristic width, emerge during the slow drainage of a liquid/granular mixture in a tilted Hele-Shaw cell. A transition from vertical to horizontal alignment of the finger structures is observed as the tilting angle and the granular density are varied. An analytical model is presented, demonstrating that the alignment properties is the result of the competition between fluctuating granular stresses and the hydrostatic pressure. The dynamics is reproduced in simulations. We also show how the system explains patterns observed in nature, created during the early stages of a dyke formation.

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I. INTRODUCTION

Subsurface flows tend to converge on high-conductivity pathways such as rock fractures, joints and faults. Flow of oil and gas in fractured reservoirs, groundwater transport, magma flow and pollutant transport in fractured porous media are therefore often dominated by the interactions between the flowing fluids, the confining geometries, and granular rock fragments residing in the cracks or faults.

A range of flow patterns can emerge when one fluid displaces another fluid in such confined spaces [1]. These flow patterns are caused by the interplay between different stabilizing and destabilizing effects, like surface tension, gravity, pore size fluctuations, wettability properties and granular effects. Viscous fingering is a well-known example of a fluid flow instability. An initially straight interface between two immiscible fluids of different viscosities develops undulations that grow to form fingers when the less viscous fluid invades the more viscous host fluid [2, 3]. In rough fractures or a porous medium, disorder in the form of variations in pore sizes perturbs the invading interface, generating fractal two-phase flow structures with no intrinsic length scale [4–7].

Gravity has a profound effect on the flow patterning in situations where a density difference between the fluids exists, and where the flow geometry is not strictly horizontal. For example, in density driven convection, the interface between a dense fluid overlying a less dense fluid becomes unstable, with dense fluid fingers sinking and low density fingers rising (the Rayleigh-Taylor instability [8]). With the less dense fluid on top on the other hand, the hydrostatic pressure stabilizes the interface at a given height. During slow drainage of a porous medium, a competition exists between the stabilizing effect of gravity, and the pore scale disorder that increases the roughness of the invasion front [9, 10].

Rock fractures and other high permeability flow paths can be filled with granular debris and fault gouge from cataclastic processes and erosion [11, 12], or materials carried by fluid flow. Multiphase flows involving both a combination of different fluids and a loose packing of granular materials have proved a particularly rich vein of pattern formation as frictional fluid dynamics is added to the well-known two-phase flow mechanisms [13]. Recently observed flow patterning processes include multiphase fracturing of deformable granular packings [13–20], decompaction fingers [21, 22], frictional fingers [23, 24] and bubble formation [13, 25].

Here we study the stabilizing effect of gravity on a patterning flow, as air displaces a

liquid/granular mixture during drainage of a Hele-Shaw cell at shallow tilt angles. The receding interface accumulates a front of granular material, and an instability caused by a competition between capillary and frictional forces results in an emerging pattern of frictional fingers – canals of air separated by branches of compacted grains [13, 23, 24]. The symmetry breaking by gravity on the tilted system causes a stabilization of the drainage front and a resulting directionality and alignment of the finger structures.

We find that the key to the finger alignment direction is a competition between gravity and fluctuations of the inter-granular stresses. Analogous to drainage in porous media [10], random fluctuations in threshold pressures cause a disruption of the stabilizing effect of gravity. However, unlike porous media, there is in our system a spontaneous emergence of a *characteristic length*, the finger width, 2Λ (Λ denotes half the finger width). The magnitude of the disruption of the invasion front becomes a relative quantity with respect to this length scale. We show that the basic assumption that the effective granular friction stresses at the interface arises as a sum of a set of uncorrelated random contributions, is sufficient to give a theoretical prediction of the transition between the different pattern morphologies.

We also show how the pattern forming mechanism provides a new understanding of the small-scale flow properties during magmatic dyke formations, i.e. the penetration of a sheet of magma into a fracture of a pre-existing rock body. The small-scale flow properties during this formation, when magma interacts with the host rock, is largely unknown [26], as the formation occurs deep beneath the Earth’s crust. Rock faces in the Israeli desert [27, 28] display aligned finger structures which were formed during a dyke formation. The structures have previously been attributed to viscous fingers, due to the Saffman-Taylor instability [2], between the fluidized host rock and a less viscous dyke-related fluid in front of the invading magma [28]. We hypothesize here, that intergranular frictional forces between quartz grains in the fluidized host rock, and not viscous forces of the fluids, govern the formation of the pattern.

II. THE EXPERIMENT

Consider a rectangular $200 \times 300 \text{ mm}^2$ Hele-Shaw cell with a gap spacing $h = 0.5 \text{ mm}$ [Fig. 1 (a)]. The cell is sealed along the sides and base; the upper end is open to the ambient air. In preparation for the experiment a granular material suspended in a 50% (by

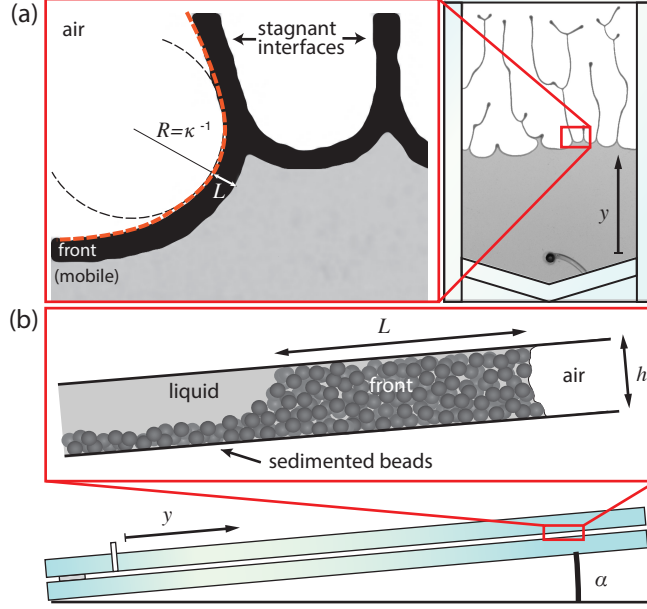


FIG. 1. (Color online) (a) Top view of the Hele-Shaw cell. The coordinate y is running from the outlet towards the upper edge of the cell, κ is the curvature (inverse of the in-plane radius of curvature R) along the interface (orange dashed line). The front is a region of accumulated grains along the air-liquid interface; L is the thickness of this front. The cell is $20 \times 30 \text{ cm}^2$. (b) Side view. The cell is tilted by an angle α . The cell gap is $h = 0.5 \text{ mm}$. The filling fraction ϕ is the height of the initial sedimented granular layer relative to h .

volume) water-glycerol mixture is injected into the horizontal cell through an inlet/outlet hole close to the base of the cell. Excess mixture spills through the open edge such that the granular suspension fills the entire cell. The granular material—spherical glass beads with mean diameter $80 \pm 10 \text{ }\mu\text{m}$ —settles out of suspension, forming a layer of grains resting on the lower glass plate of the cell. The height of this layer, relative to the cell gap, is denoted ϕ , and quantifies the initial filling fraction of the injected granular mixture relative to the random loose packing fraction of the grains. The glass beads are polydisperse, and the variation in size prevents crystallization of the sedimented bead packing. The density of the glass beads and the water-glycerol mixture is $\rho_g = 2.4 \text{ g/cm}^3$ and $\rho = 1.13 \text{ g/cm}^3$ respectively. The bead-fluid density contrast makes the beads sediment on the bottom plate.

The long side of the cell is tilted by an angle α relative to the horizontal plane [Fig. 1 (b)]. Here we report only results for shallow tilt angles ($0^\circ \leq \alpha \leq 5^\circ$) where no sliding of the granular layer takes place. The experiment commences by slowly draining fluid from

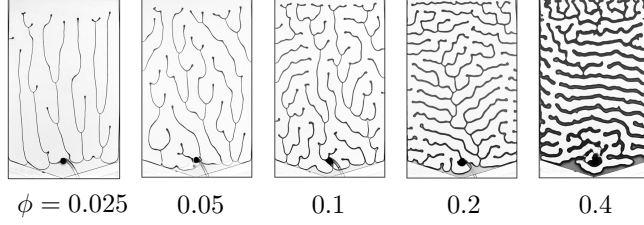


FIG. 2. Final configuration of the experimentally observed pattern at constant tilt angle, $\alpha = 4^\circ$, with varying filling fraction ϕ . The finger alignment changes direction from vertical to horizontal as ϕ increases. Each image frame is 200 mm wide.

the outlet at the base at constant withdrawal rate $q = 0.07$ ml/min, using a syringe pump (WPI, Aladdin 1000). The withdrawal rate is slow enough to leave the layer of grains along the air/fluid interface undisturbed by the fluid flow. As fluid is slowly drained, air starts to invade, and the meniscus along the elevated open edge of the cell gradually recedes. The system is imaged from underneath using a PL-B742U Pixelink camera, and illuminated by a white screen placed above. Compacted granular material appears dark in the images, and empty regions of the cell appear white.

As the air displaces the liquid, grains accumulate along the air-liquid interface and fill the cell gap, forming a dense pack in a region adjacent to the interface which we refer to as the *front* (see Fig. 1). Only a small section of the interface moves at any given time, and the motion consists of incremental displacements, as the air fills an ever-increasing volume. A moving section tends to continue its motion over many consecutive increments before it stops and the motion continues at another section. The interface develops frictional fingers of air surrounded by a front [13, 23, 24], with a characteristic finger width. When different fingers move towards each other, their fronts combine, and their interfaces stagnate. The evolution continues until either the whole cell is filled with air and stagnant fronts, or the air reaches the outlet.

When the cell is fixed horizontally ($\alpha = 0^\circ$), the finger directions are disordered and isotropic, and the resulting patterns are labyrinth structures of stagnant fronts [23, 24]. When the cell is tilted, the frictional fingers tend to align [29]. The direction of alignment changes as we vary α or ϕ . Fig. 2 shows the residual patterns of granular material in the shape of narrow branches after all the grains have been packed at the end of each experiment. The figure displays results from a series of experiments with increasing filling fraction, with the

tilt angle kept constant at $\alpha = 4^\circ$. The pattern of residual granular material bears witness to the dynamics of the invasion process. At low ϕ , the air fingers march downwards, from top to bottom, leaving granular branches aligned with the direction of gravity (vertical in the images). At high ϕ , the system makes a transition to sideways growing air fingers, leaving a trail of horizontally aligned granular branches. Alternatively, by keeping ϕ constant, and increasing the tilt angle from 0° to 5° , it is possible to go from random labyrinthine pattern to horizontal alignment and then to vertical alignment at high α .

In the low ϕ /high α range, hydrostatic height stabilization of the receding interface dominates the dynamics, the fingers advance side-by-side downwards, parallel to the gravitational field along the cell [Fig. 3 (a), SM Video 1 [30]]. Lateral growth is inhibited by the presence of neighboring fingers on both sides; each finger is confined to downwards growth. A finger will terminate its movement if it is bypassed and sealed off by its neighboring fingers. A finger can also split in two if a small region along the finger tip gets stuck, and each side of this region evolves to separate fingers. This typically happens when a finger tip widens, which seems to happen in conjunction with the termination of a neighboring finger. Finger termination and tip-splitting occur at approximately equal frequencies [see Fig. 3 (a)]. We note that these patterns look remarkably similar to patterns generated when simulating retraction of a dewetting suspensions [31], although the setup is completely different.

As we increase ϕ and reduce α , we observe a gradual transition in the alignment; the fingers tend to grow with a directional component transverse to the hydrostatic pressure gradient. In the intermediate range of ϕ and α , hydrostatic stabilization of the front occurs, but local pressure fluctuations enable some fingers to get ahead. Sideways growth is preferred for a finger that extends beyond its neighbors due to the hydrostatic pressure gradient. The finger which manages to get ahead fills a larger fraction of the horizontal direction, and advances layer by layer, creating a pattern of horizontal lines (Fig. 3 (b), SM Video 2 [30]). In the high ϕ /low α range the local pressure fluctuations dominate over the stabilizing effects, and alignment is lost. A phase diagram of the alignment behavior of the end configurations is shown in Fig. 4.

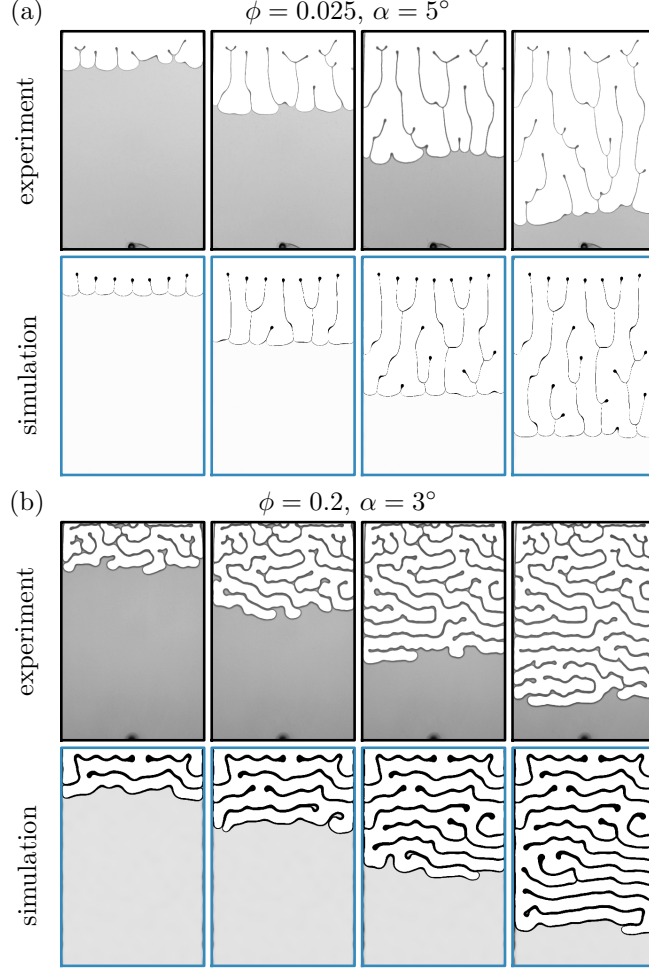


FIG. 3. (Color online) Snapshots of the dynamics, experiments versus simulations. (a) The pattern is dominated by vertically aligned fingers at $\phi = 0.025$ and $\alpha = 5^\circ$. (b) The pattern is dominated by horizontally aligned fingers at $\phi = 0.2$ and $\alpha = 3^\circ$. See (a) SM Video 1 and (b) SM Video 2 [30].

III. MODEL

As the dynamics are manifested by incremental movements of confined regions of the interface, it is reasonable to assign a yield pressure threshold to every point along the interface. When the pressure difference at the interface exceeds the threshold at the weakest point along the interface, the interface locally to that point deforms and moves a small step towards the liquid phase. This approach has successfully modeled labyrinth patterns in a similar setup [23, 24], but without considering the hydrostatic pressure differences induced by the tilting of the cell. In order to quantify the yield pressure threshold, we will assign two local parameters to the interface: the front thickness L and the apparent in-plane curvature

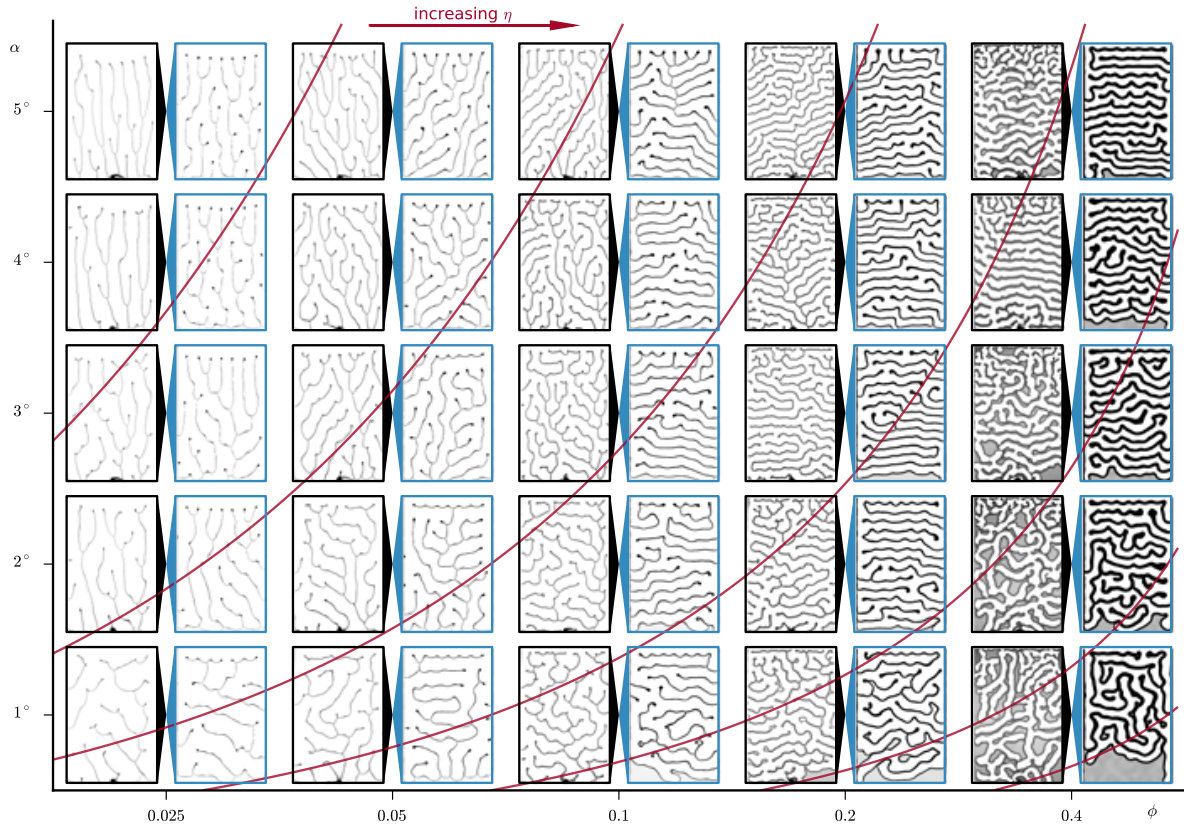


FIG. 4. (Color online) Pairwise comparison of the final configuration of experiments (black/left frames) to simulations (blue/right frames), for different values of the filling fraction (ϕ), and the tilting angle (α). The red lines indicate contours of constant η which are estimated up to a constant factor in Eq. (9). As η increases, the vertical alignment turns into horizontal alignment, and then into no alignment. The value of η doubles for every contour. The gravitational pull is pointing downwards in every frame.

κ .

The front thickness, L , is the distance from the air-liquid interface, in the perpendicular direction, to the region of the liquid mixture where the beads no longer fills the whole cell gap (see Fig. 1). Note that the packing of beads in the front remains in a static configuration before a potential movement. We assign a yield stress $\sigma_Y(L)$ to every point along the interface, which captures the static frictional properties of the front. To be precise, σ_Y is the yield stress acting normal to the plane which approximate the air/liquid interface. This yield stress has previously, in the context of labyrinth patterns [24] and of plug formations in narrow tubes [32], been assumed to be exponentially increasing in front thickness L . The

exponential behavior can be justified by considering Janssen’s model for stresses in packings of grains, which assumes a linear relationship between the principal stresses in the packing, in conjunction with the static Coloumb frictional stresses at the plate boundaries of the cell. The yield stress may also have a curvature dependence, as described in Ref. [25]. In the following, we will, however, describe the yield stress as a linear function in L ,

$$\sigma_Y(L) = \frac{\sigma_\xi}{\xi} L, \quad (1)$$

for simplicity. The numerical comparison to the experimental behavior in the subsequent section, will validate this approximation as sufficient for the range of parameters that we consider here. The expression in the equation above has two interpretations. We can interpret it as a linearization of a more complicated function of L , e.g. the exponential behavior assumed in [24, 32]. Alternatively, we can interpret the yield stress as a sum of consecutive force bearing arc chains [33] which transmit frictional stresses, σ_ξ , from the cell plates to the beads at the air-liquid interface. The characteristic length of these chains is ξ , and the total number of chains scales with the size of the front and therefore linearly in L .

The air-liquid surface tension at the interface acts at two different scales. At the small scale, the interface makes bridges between wetting beads. Each point on a meniscus can be characterized by two principal radii of curvature. By the Young–Laplace equation, the pressure drop over a meniscus is proportional to the mean of the principal curvatures. This means that in a static configuration, each meniscus has the exact same mean curvature, up to differences in the hydrostatic liquid potential, which we can ignore in a horizontally oriented cell.

At a larger scale, we can identify a curvature which is averaged over several neighboring beads. For our Hele-Shaw setup, the principle directions of the average curvature are the in-plane and the out-of-plane directions with respect to the cell plane. We will disregard the curvature component in the out-of-plane direction of the cell, i.e. the curvature of the interface as it is illustrated in the cross section in Fig. 1 b. The out of plane curvature is supposed roughly constant, i.e. the surface stress related to this component is constant along the in-plane direction of the interface, and does, at our level of description, only contribute to a constant global pressure drop. It plays no role when we later need to determine the minimal yield stress.

The large scale surface behavior, i.e. the surface behavior averaged over many neighboring

inter bead menisci, can be characterized by an effective surface tension γ [24]. The effective tension acts against the increase of the apparent interface area during the displacement process, and the associated pressure difference is simply $\gamma\kappa$.

We can now quantify the local yield pressure threshold. Let Δp be the difference between the air pressure, p_{air} , which is considered constant, and the liquid pressure at the outlet of the cell, p_{outlet} . We assume that a section of the interface is mobilized if

$$\Delta p \geq \gamma\kappa + \frac{\sigma_\xi}{\xi}L - y\rho g \sin \alpha. \quad (2)$$

The first and second terms on the right hand side is the effective surface stress and the yield stress [Eq. (1)] described above. The last remaining term is the hydrostatic pressure relative to the base of the cell, y is a coordinate running along the cell from the outlet, g is the gravitational acceleration and ρ is the liquid density. This amounts to say that the local pressure in the fluid behind the meniscus, $p_{\text{air}} - \gamma\kappa$, is equal to the sum of the solid and the fluid stress there. The fluid stress there is $p_{\text{outlet}} - y\rho g \sin \alpha$. Hence, the solid stress there is $\sigma_{\perp\text{solid}} = p_{\text{air}} - (p_{\text{outlet}} - y\rho g \sin \alpha) - \gamma\kappa$. If the solid stress is equal or larger than $\sigma_\xi/\xi L$, the grain pack slides locally. The pressure difference, Δp , will increase when the whole interface remains static and liquid is drained from the system. The next moving section, at any given time, is identified by local parameters κ , L and y , which minimizes the right hand side of Eq. (2). As the section yields and moves a small step towards the liquid, the local parameters are changed due to the deformation and the accumulation of new beads onto the front.

A. Numerical Validation

We can reproduce the experimental behavior in a numerical simulation. The numerical scheme has previously been used to simulate finger behavior in a flat cell [33]. We present here a summary of the numerical strategy, and the modifications which are needed for the tilting of the cell. Further details of the numerical scheme are described in Ref. [33].

The fluid interface (i.e. the boundary of the gas phase), can be represented as a chain of nodes, labeled by an index i , where each node carries information of the spatial coordinates (x_i, y_i) , and its nearest neighbors, $i \pm 1$. Such a chain can conveniently be implemented like a doubly linked list. We couple this chain of nodes to a two dimensional mass field,

representing the grains. The complete filling of the cell gap, i.e. the region which constitutes the front, is indicated by the region of the mass field which exceeds a threshold value. We make sure that the region of the mass field adjacent to the chain, i.e. the region of the front, exceeds this threshold in the initial configuration of the system. The imposed dynamics described below will maintain this state.

For each node we can identify the two local properties. First, the local front length L_i is represented as the shortest distance from any given node, to a cell in the mass field which take a value below the threshold. This cell will be referred to as the *link* cell associated to the node. Second, we can approximate the local curvature, κ_i , at node i , by numerical differentiation of a spline approximation of the nearest and next nearest neighbors $\{i, i \pm 1, i \pm 2\}$. By discretizing the right hand side of Eq. (2), we can now identify a pressure threshold T_i for each node,

$$T_i = \gamma\kappa_i + \frac{\sigma_\xi}{\xi}L_i - y_i\rho g \sin \alpha. \quad (3)$$

The dynamics of the system is generated by iteratively moving the node with the minimal value of T_i , an infinitesimal distance towards the fluid phase, in the perpendicular direction to the interface. At each step we need to accumulate new beads from the initial distribution to the front. This can be achieved by adding the gathered bead mass which corresponds to the infinitesimal displacement, to the link cell of the node. If this cell reaches the threshold value, a new link cell will be assigned, and the rest mass will be distributed there. This approach will make sure the bead mass field is conserved. The chain is interpolated with new nodes as the interface grows, keeping the resolution of the representation of the interface constant, and the local quantities, κ_i and L_i , are recalculated in a neighborhood along the chain near the moving node.

Note that there is no time in this numerical approach. We can, however, estimate the time from the volume of the air phase, as we know that the drainage rate q is constant. This allows us to compare the experimental results to the numerical simulation during the evolution of the patterns. The dynamics is deterministic, and the random behavior is a result of perturbed initial conditions, and imposed quenched fluctuations in the initial mass field. Note that the random fluctuations in the mass field will induce fluctuations in L_i , as mass is accumulated. These fluctuations scale with $\sqrt{L_i}$ as L_i correspond to a sum of multiple randomly distributed masses. This effectively induces fluctuations in T_i evaluated

at each node.

We use $\sigma_\xi/\xi = 16 \text{ kPa/m}$, which is an estimate based on comparison between experimental results and the theoretical expression for finger width [33]. For the effective surface tension we use $\gamma = 60 \text{ mN/m}$ [24]. The similarity between the simulated and experimentally observed patterns (Fig. 4) validates our theoretical understanding. A noticeable difference between simulations and experiments is that liquid pathways in the front may break, resulting in isolated pockets of liquids in the experiments at high ϕ /low α . These effects are not accounted for in the simulation, as we locally only track the interface and the grains, not the fluids.

B. Transition of alignment direction

To understand the transition between horizontally and vertically oriented finger behavior, we need first to quantify the variations in the yield pressure threshold [Eq. (2)]. It is hard to quantify the exact numerical value of these variations, but it will suffice for our purposes to determine how the variations scale with L . We will assume that these variations are dominated by the variation of the static friction along the front [Eq. (1)]. If we interpret Eq. (1) to be a sum of force bearing arc chains of length ξ , each of which contributes with a varying yield stress with a mean value of σ_ξ , then the total variation will scale with the number of these chains. As the number of chains scales with the size of the front, we have that $\text{Var}(\sigma_Y) \propto L$, and that the standard deviation is proportional to \sqrt{L} . Note that in the numerical simulations the value of σ_ξ is kept fixed (it is not a random variable), but we introduce another physical source of fluctuation leading to the same scaling. We impose the fluctuations in the initial bead density field, which induces fluctuations in the front thickness that also scale with \sqrt{L} for a fixed displacement of the interface.

We can compare these variations to the hydrostatic pressure difference over a horizontally oriented finger. The finger width is 2Λ , and the corresponding hydrostatic difference is $2\Lambda g \rho \sin \alpha$. The ratio between the standard deviation of the yield stress, and the hydrostatic difference of a horizontally oriented finger is therefore,

$$\eta \propto \frac{\sqrt{L}}{\Lambda \sin \alpha}. \quad (4)$$

This ratio indicates the behavior of the alignment. When the contribution of stress fluctua-

tions is comparable to the stabilizing pressure ($\eta \simeq 1$) a finger can get ahead of its neighbors and grow sideways, orthogonal to the direction of gravity. For $\eta < 1$, the fluctuations fail to disrupt the side-by-side finger growth. For $\eta > 1$, the fluctuations dominate over the stabilizing effect, and the alignment is lost. We can only estimate η up to a multiplicative constant, as the numerical value of the stress variations of σ_Y is hard to identify. This will, however, suffice for identifying the contour lines in the (α, ϕ) plane, which have similar alignment properties. To identify these contour lines, we first need to express Λ and L in terms of ϕ .

Let A and C be respectively the area and the circumference of the air phase, as seen from above, and let h be the cell gap. The pattern is dominated by finger structures, such that $A = C\Lambda$. We assume that L is approximately constant along the interface, such that CL is the total area of the front. Mass conservation gives that $h(CL + A)\phi = hCL$, which under the substitution $\Lambda = A/C$, implies that

$$L = \Lambda \frac{\phi}{1 - \phi}. \quad (5)$$

A more detailed derivation, which differentiates between the front thickness at the sides and the tip of the fingers, yields correction terms to this expression (see Ref. [33]).

The work of a typical displacement, δw , has two contributions when we set $\alpha = 0$ for simplicity. First, the stretching of the interface contributes with $\gamma h \delta C$, where $\delta C = \delta A/\Lambda$, which follows from the assumption of constant Λ . Second, the work done against the granular stresses, σ , in the front, is $hs \delta x \sigma$, where s is the typical width of a moving segment and δx is the distance the interface advances such that $s \delta x = \delta A$. We can approximate σ , by the yield stress, σ_Y [Eq. (1)]. Putting the terms together, and dividing by the displacement duration, gives the work rate,

$$\frac{\delta w}{\delta t} = \left(\frac{\gamma}{\Lambda} + L \frac{\sigma_\xi}{\xi} \right) h \frac{\delta A}{\delta t}, \quad (6)$$

where $h \delta A / \delta t$ equals the constant compression rate, when averaged over many stick-slip events. Substituting Eq. (5) and minimizing Eq. (6) with respect to Λ gives

$$0 = \frac{d}{d\Lambda} \frac{\delta w}{\delta t} \Rightarrow 0 = \frac{d}{d\Lambda} \left(\frac{\gamma}{\Lambda} + \Lambda \frac{\phi}{1 - \phi} \frac{\sigma_\xi}{\xi} \right), \quad (7)$$

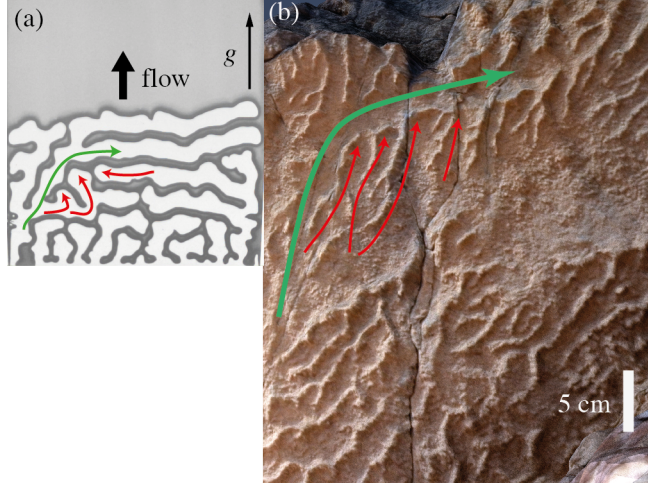


FIG. 5. (Color online) Feature comparison 1 between the experimental observations at $\phi = 0.4$, $\alpha = 4^\circ$ (a) remaining structures on dyke walls found in the Inmar formation (b). Fingers (red arrows) are being intercepted by a finger (green arrows) which grows perpendicular to the average flow direction. The gravitational pull is indicated by g . The scale bar in (b) applies to both experiment and dyke figure.

which corresponds to the assumption that the pattern evolves in a way that minimizes the work. This implies

$$\Lambda \propto \sqrt{\frac{1-\phi}{\phi}}. \quad (8)$$

We can now use Eqs. (5) and (8) to rewrite Eq. (4) as a function of ϕ and α ,

$$\eta \propto \frac{1}{\sin \alpha} \left(\frac{\phi}{1-\phi} \right)^{3/4}. \quad (9)$$

Indeed, contours of constant η correspond to equal qualitative alignment behavior, as shown in Fig. 4.

IV. APPLICATION: FLOW IN DYKES

We now turn to the relevance of this system to magmatic flow during dyke formations. A (magmatic) *dyke* is an approximately two-dimensional sheet-like body of magma, which has penetrated a pre-existing body of rock in a direction which is perpendicular to the bedding planes (i.e. the planes of the sediments). A striking example of a dyke formation is

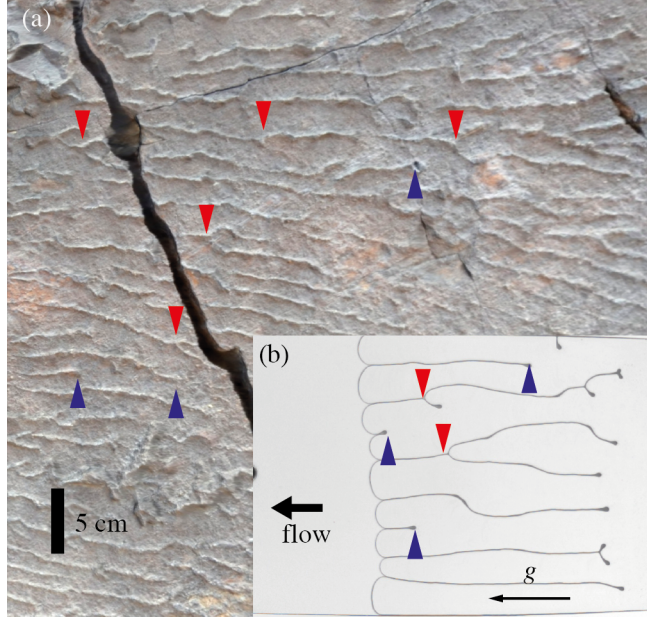


FIG. 6. (Color online) Feature comparison 2 between the experimental observations at $\phi = 0.025$, $\alpha = 4^\circ$ (b) and the remaining structures on dyke walls found in the Inmar formation (a). Aligned finger structures with tip-splitting and termination, respectively marked by blue and red triangles. The gravitational pull is indicated by g . The scale bar in (a) applies to both experiment and dyke figure.

found in the Inmar formation in the desert in southern Israel. There, the igneous rock (i.e. the solidified magma) of the dyke has eroded away, and the erosion resistant dyke walls, made by quartzitic sandstone, are exposed. These walls display a rich network of finger structures [27, 28]. The fingers are identified as grooves in the sandstone; outward bulging ridges separate the fingers from its neighbors. A finger is approximately 1-10 cm wide and 10-100 cm long, and the wall shows intermittent patches of finger alignment [Figs. 5 (b) and 6 (a)]. The walls are separated $\simeq 1$ m apart, but mirror images of the structures remain on both walls, which suggests that the structures were made during the initial stages of the dyke formation.

The ridges contain a closely packed concentration of quartz grains (100-500 μm diameter) cemented by iron oxides and kaolinite, in contrast to the relatively low concentration of quartz grains (similar in composition) in the rock near the grooves (see Fig. 12 in Ref. [28]). The finger structures indicate that the sandstone was fluidized during the formation while these quartz grains were preserved as solid, and that the grains were accumulated onto

stagnant regions adjacent to the interface of the invading magma, which initially filled the grooves.

The finger formation in these dykes has previously been explained as viscous fingers due to the potential viscous contrast between an invading dyke-related fluid and the fluidized host rock [28]. Viscous fingers in porous media are, however, known to display fractal invasion patterns with no intrinsic length scale [4–6], whereas the fingers on the dyke walls display a characteristic width. The similarity of these dyke wall fingers to the aligned finger structures observed in our experimental setup, suggests that the fingers are generated by inter granular friction between the quartz grains and accumulation of these grains onto stagnant fronts.

The relevance of our system to the structure in the Inmar formation is further substantiated by the similarity in the features of the resulting pattern. In particular we observe similar tip-splitting and termination properties [Fig. 6], and interception of fingers by a finger which grows perpendicular to the average flow direction [Fig. 5].

The fingers direction in the Inmar formation varies locally between vertically upwards and downwards. Steps in the dyke structures indicate that the intrusion was following a propagating crack [28]. The consistent direction of the crack direction, and the vertical orientation of the dykes, indicate that no large geological deformation took place after the formation. The direction of the gravitational effect on the fingers, depends on the density contrast of the invading fluid to the fluidized host rock, which is unknown. Variations in the crack opening will, however, induce a stabilizing potential in the capillary pressure as the out-of-plane component of the magma interface curvature increases towards the crack tip. A combination of hydrostatic and capillary pressure variations, is therefore likely to act as the stabilizing potential in the dyke finger formation. Variations in the crack spacing also explain the local variations in the finger directions, and the presence of features [Figs. 5 (b) and 6 (a)], from different parts of the phase diagram [Fig. 4 (c)].

V. CONCLUSION

We have described a new type of pattern forming flow, where grains are accumulated by a moving interface, which, when subject to a stabilizing potential, forms aligned finger structures. We identify the finger width by a work minimization principle, and can estimate the alignment direction by the competition between frictional force fluctuations and the

hydrostatic pressure. The dynamics is quasi-static; it depends on granular friction rather than viscosity. The patterning process seems to be independent of whether the invading fluid is a gas or a liquid, as long as the phases are immiscible. We can reproduce the finger behavior numerically by accounting for the hydrostatic pressure, grain accumulation, solid friction and capillary forces. As our model only contains geologically ubiquitous mechanisms, it may be relevant for a number of biphasic flow phenomena confined to planar fractures, in particular multiphase flow during dyke formation that leave imprints of the finger formation as solidified granular residue on the dyke walls.

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